Winsight® Assessment Mathematics Learning Progressions

DATA DISPLAY

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Mathematics Learning Progressions: Data Display

What is the Data Display Learning Progression?

The visualization of information and data in graphical representations is prevalent in both scientific practices and everyday life situations. The learning progression for Data Display focuses primarily on how the understanding of data display develops and deepens in middle-school grades, with a focus on graphing data and reading statistical graphs. We define statistical graphs as graphical representations of statistical data (e.g., pictographs, dotplots, bar graphs, pie graphs, scatterplots, histograms), excluding tables or other symbolic formats to display data. Students may use this knowledge and these skills in the real world when they read news reports or communicate statistical information to others. We have highlighted two main ideas, otherwise known as progress variables, in this learning progression: a) constructing statistical graphs, and b) interpreting statistical graphs. The American Statistical Association® recommends that both of these activities should occur in the process of meaningful statistical inquiry because "data are not just numbers, they are numbers with a context" (p. 7).[1]

Connecting The Data Display Learning Progression to the Common Core State Standards

The Data Display learning progression connects to several Common Core State Standards for mathematics for Grades 6 to 8, such as 6.SP.4 (Display numerical data in plots on a number line, including dot plots, histograms, and box plots) and 8.SP.1, 2, and 4 (Investigate patterns of association in bivariate data). This learning progression also describes earlier expectations, such as 4.MD.4 (Represent and interpret data) and can be extended up to the High School Statistics and Probability Standards. Several of the Standards for Mathematical Practice can be addressed when acquiring knowledge and skills in Data Display, such as MP.4 (Model with mathematics) and MP.6 (Attend to precision).

What are the Levels of the Data Display Learning Progression?

The Data Display learning progression has two progress variables that cut across all levels: Constructing Data Display and Interpreting Data Display. While Constructing Data Display is presented before Interpreting Data Display, research does not indicate that one develops before the other. In fact, the development is most likely symbiotic.[2]

The progression (Table 1) is shown in from Level 6 (at the top) to Level 1 (at the bottom) to draw attention to the growth within the progression.

[1]Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M. & Schaeffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. Alexandria, VA: American Statistical Association. Available at: https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf
Moore, D. & Cobb, G. (1997). "Mathematics, Statistics, and Teaching," *American Mathematical Monthly*, 104, 801–823
[2]Edwards, T. G., Özgün-Koca, A., & Barr, J. (2017). Interpretations of boxplots: Helping middle school students to think outside the box. *Journal of Statistics Education*, 25(1), 21-28.

Table 1: The Data Display Learning Progression

	Constructing Data Display	Interpreting Data Display
Level 6	**Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data.** (1) Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data, namely moving back and forth between the two perspectives according to a given graphing goal and context. (2) Students construct statistical graphs using proportional reasoning skills. (3) Students understand the purpose, use, and limits of different forms of statistical graphs to represent different types/nature of data.	**Students interpret statistical graphs from the integrated view of data relations to make predictions about general trends of data values.** (1) Students interpret statistical graphs with an integrated view of case-oriented and aggregate views of data, namely moving back and forth between the two perspectives according to a given graphing goal and context. (2) Students interpret statistical graphs using proportional reasoning skills. (3) Students use central tendency and variability to make predictions while interpreting statistical graphs. (4) Students provide summary comments about the data represented in statistical graphs.
Level 5	**Students construct statistical graphs with an aggregate view of individual cases.** (1) Students quantify aggregate properties (e.g., ratios, portion, or percent) of data values in statistical graphs. (2) Students are able to construct statistical graphs associated with a bivariate data set. (3) Students are able to develop additional statistical graphs to represent a (univariate or bivariate) numerical data set.	**Students interpret data relationships displayed in statistical graphs to infer the relationships implicit in data values.** (1) Students describe the shapes of data distributions presented in statistical graphs through quantification of aggregate properties (e.g., ratios, portion, or percent) of data values. (2) Students identify trends in data values (e.g., increases, decreases, or fluctuations) through visual comparison of specific graphical features represented in statistical graphs, and then make inferences about data relationships that are not directly presented in the graphs. (3) Students use proportional strategies in visually comparing two groups of different sizes presented in statistical graphs.
Level 4	**Students construct statistical graphs while attempting to aggregate individual cases to address graphing goals.** (1) Students start to consider aggregation of individual cases, such as grouping similar data values within an interval or range. (2) Students begin to construct statistical graphs with a conventionally graded scale for major divisions (e.g., fives or tens) to meet a given goal or context. (3) With regard to the selection and usage of appropriate graph form for the nature of a given data set, students are able to identify appropriate types of statistical graphs to represent numerical or categorical data (e.g., drawing a bar graph for categorical data and a histogram for numerical data).	**Students analyze information presented in statistical graphs to identify the relationships among data values.** (1) Students identify data relationships by integrating information across two or more data values shown on statistical graphs. (2) Students begin to see the shape of data with regard to frequency, order, interval, and/or scale of data (values) represented on statistical graphs; they attend to the shapes that these distributions take (e.g., symmetric or skewed) and can identify the interval having most data values. (3) Students are able to provide an appropriate summary of the data presented on statistical graphs, commenting on the symmetry of the graphs. (4) Students identify data values that are different between two graphs, and then use a numerical comparison strategy (e.g., calculating the sum of the data values of each group) in comparing two groups of equal size.

Table 1: The Data Display Learning Progression

Level 3	**Students draw statistical graphs associated with individual cases (i.e., with a case-oriented view of data).** (1) Students construct graphical components that are appropriate for the given graph type and use scaling and labeling correctly, but the degree of success may depend on the complexity of given data. (2) Students create statistical graphs of individual cases, showing insufficient data reduction. (3) Students create inappropriate statistical graphs for the nature of a given data set (e.g., drawing a bar graph to display numerical data values).	**Students interpret graphical features shown on statistical graphs to find the relationships between/among data values.** (1) Students interpret particular graphical components shown on statistical graphs to meet a given graphing goal or context. (2) For data values or quantities directly presented on statistical graphs, students identify data relationships of numerical difference, relative difference, and proportions through more than one strategy. (3) Students attend to individual data values or quantities in comparing two groups of equal sizes.
Level 2	**Students create informal, unconventional statistical graphs with partial recognition of scale; namely, students are at least consistent in their representations even though they are not using standard scales and/or visuals.** (1) Students engage in graphing data values to capture the shape of data that they perceive in a data set. (2) Students draw informal, unconventional statistical graphs using labeled or unlabeled components.	**Students read statistical graphs to locate specific data values with insufficient attention to a given graphing goal or context.** (1) Students begin to connect data values to graphical components (e.g., the bars of a bar graph). (2) Students can locate the most salient data values (e.g., maximum, minimum, or middle value), with no attention to central tendency or variability of the data values represented on the graph. (3) Students misread some aspects of statistical graphs since they attend to salient data values only and use intuition to complete their interpretation. (4) Students give non-statistical reasons for trends in data values.
Level 1	**Students create idiosyncratic statistical graphs.** (1) Students perceive that data values can be displayed graphically. (2) Students draw idiosyncratic statistical graphs without attending to a given goal or engagement with a given context. (3) In drawing idiosyncratic statistical graphs, students rely on their personal beliefs and experience.	**Students read statistical graphs idiosyncratically.** (1) Students perceive that statistical graphs show a series of data values. (2) Students read statistical graphs without attending to a given goal or engagement with a given context. (3) In reading statistical graphs, students rely on their idiosyncratic ideas, such as personal beliefs and experience.

At Level 1 of the Data Display learning progression, students create and read graphs idiosyncratically, often relying on their personal beliefs and experience, instead of data values, to make and interpret graphs. At Level 2, students begin to engage in more structured graphing by constructing statistical graphs using an unconventionally graded scale or using different-sized graphical components for one type of independent, categorical variable (e.g., using different-sized "boxes" for the bar of frequency in a bar graph). As they begin to read graphs, students performing at this level can identify the most salient components of a graph but may misread other components. The knowledge and skills in Levels 1 and 2 are likely to be found in the work of elementary-school students.

At Level 3, students begin to use conventional scales and appropriate graph types. In constructing bar graphs, for example, the x- and y-axes are appropriately constructed and labeled; groups of categorical variables are structured on the x-axis and quantitative values are structured on the y-axis; and the magnitude of the quantitative values on the y-axis is represented by the heights of bars with a conventionally graded scale. In interpreting such graphs, students can identify numerical differences, such as numerical comparisons of sums, means, or visual comparison of part-to-part or part-to-whole.

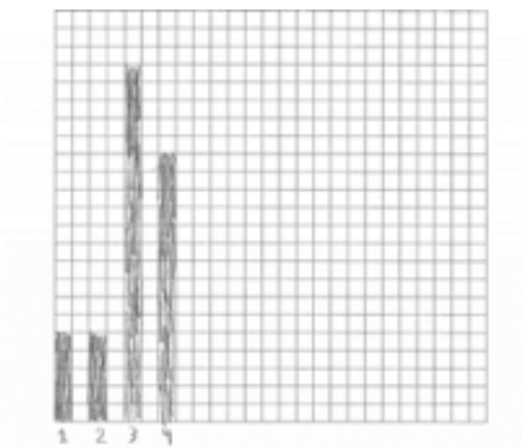
Students at Level 4 become more aware of aggregated cases, whether by attending to the shape of the data distribution, the graphing goal, and/or context. In constructing graphs, students consider the nature of the data and the graphing purpose when choosing types of graphs to produce. In interpretation, students identify relationships in the data and are able to summarize the information presented graphically. Level 3 and 4 understanding is likely to be a target of instruction in the middle grades.

The work produced at Levels 5 and 6 shows increasing sophistication in inference and working with bivariate data. Graphs constructed at Level 5 may present aggregated quantities, such as ratios or percentages. In interpreting graphs, students can infer relationships that are implicit in the data, for example, by estimating linear relationships in a scatterplot (absent a line of best fit). Students also use proportional reasoning skills (instead of or in addition to numerical difference) to compare quantities.

At Level 6, students can emphasize cases or an aggregated view of the data, depending on the graphing goal and context. Students also understand the limitations and benefits of different types of statistical graphs. In interpreting graphs, students can identify case-based or aggregate information and can use graphs to predict statistical trends. While some students may reach Level 6 during high school, many may need post-secondary instruction to do so.

What Do The Levels Of The Learning Progression Look Like In Student Work?

Below is an example of a Data Display task targeting Levels 1-6 of the progression. The learning progression can help educators anticipate and address different student responses. In this task, students are given completed surveys from 45 camp attendees responding to the question: "Among the four tasks we did today, which task was your favorite one? Please choose only one activity." Camp attendees could choose: Activity 1: Forecasting the Weather, Activity 2: Reading Math Stories, Activity 3: Building a Lego® City, or Activity 4: Programming Robots. Students are asked to create a way to organize the surveys to show how popular the activities were and to draw a graph. Then, they are asked to interpret the graph they created.



Student 1

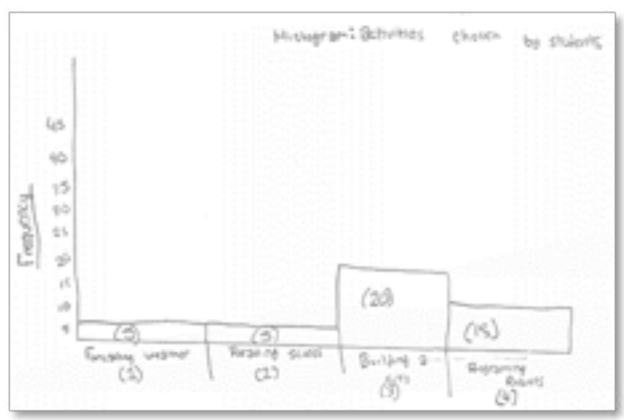
This student's graph is typical of those produced at Level 3 in *Constructing*. Although labeling of the x-axis is incomplete, the student uses a conventionally graded scale and structures the bar graph correctly. (At level 1 or 2 a student might line up all of the student activity choices along the x-axis without grouping by activity.)

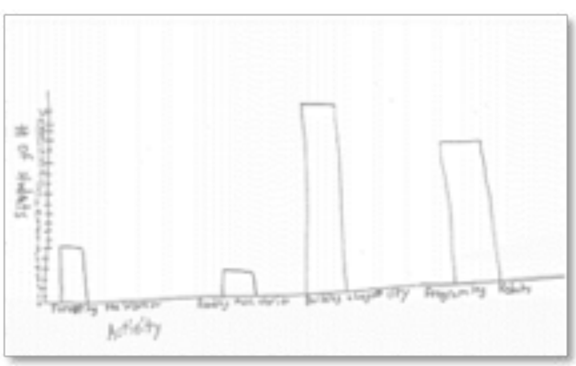
His *Interpretation* is also at Level 3. In describing the pattern in the graph, this student was able to identify the numerical differences when comparing the frequencies of camp attendees choosing Activities 2 and 4 versus Activities 1 and 3.

Student 2

Student 2 produced work that was at a higher Level in *Constructing* than in *Interpretation*. This bar graph shows evidence of Level 4 *Constructing* with its y-axis structured by 5s.

However, the work was consistent with Level 3 *Interpretation*, as the student struggled to summarize the data: "…As you can see it shows the frequency and the 4 activities. The activities had students choosing their favorite one and the frequencies represented how many students each activity had!" A less advanced interpretation might reveal misunderstandings about the variables or the data set.





Student 3

Student 3 produced work that was at a higher Level in Interpretation than in Constructing. The bar graph shows Level 3 skill in *Constructing* as it uses a conventionally graded scale and appropriate labels, but no real attempt to aggregate the frequencies by 5s is made. On the other hand, this student offers a Level 5 *Interpretation* of potential reasons for the data pattern: *"…students preferred doing things they could see right then and there…building a city or programing a robot because the results of success were right in front of them. With forecasting the weather, students had to wait for their predictions to be proven or disproven…"*

How Can We Help Students Learn?

Teachers can use the Data Display learning progression in the following ways:

- As a guide to anticipate and interpret student thinking, including selecting instructional materials or developing classroom assessments (e.g., Are the students able to describe the shape of the data distribution? Are they able to summarize the information presented in graphs? Can they choose an appropriate graph for the data and the graphing goal?)
- To develop hypotheses about what students do and do not yet understand, based on evidence of student thinking
- To determine next steps to support emerging understanding

Teachers can engage with these, and other, practices individually or with colleagues. Examining student work to understand how it addresses the standards while using the learning progression to interpret more and less sophisticated responses can support further instruction. Planning together how to give feedback to students or identify next instructional steps for students who are at different levels of the learning progression can also be a useful professional learning experience.

Students may also find the learning progressions useful with some translation into more student-friendly language and exemplars to illustrate reasoning at different levels of the progression.

For Additional Insights

Other relevant learning progressions are Variability, Probability, Mathematical Modeling (identifying when and how a situation can be modeled using data), and Argumentation (developing and critiquing arguments about a situation using data as evidence).

For More Information

For further reading, see the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*, available at: https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf

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