# *Winsight*® Assessment Mathematics Learning Progressions

## MATHEMATICAL ARGUMENTATION

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# Mathematics Learning Progressions: Mathematical Argumentation

## What is the Mathematical Argumentation Learning Progression?

Mathematical Argumentation includes making a conjecture, proving a proposition, justifying an inference, or explaining a point. Students have been found to demonstrate argumentation skills when arguing with and asking questions of peers. In addition, explanation and justification of one's own thinking such that it can be understood by a specific audience is considered a fundamental part of this mathematical practice. The learning progression for Mathematical Argumentation focuses primarily on how the understanding of argument and proof develops and deepens in elementary and middle grades (Table 1). Students may use mathematical argumentation when they encounter real-world problems such as convincing another student that their solution to a problem is correct or better than another potential response, or that two seemingly different approaches are both mathematically correct.

## Connecting the Mathematical Argumentation Learning Progression to the Common Core State Standards

The Mathematical Argumentation learning progression connects to several Common Core State Standards for mathematical practice across all grades. Mathematical argumentation is directly addressed in CCSS-MP3: "Construct viable arguments and critique the reasoning of others." There are also other practices that are closely related to mathematical argumentation, such as "Attend to Precision" (MP6), "Look for and make use of structure" (MP7), and "Look for and express regularity in repeated reasoning" (MP8).

It should also be noted that students must engage in mathematical argumentation in the context of other mathematical content. Thus, it is important to make sure that students are engaging in argumentation using content with which they are familiar enough to comprehend and produce arguments. If working with content that is at-level with student ability, we may see inconsistent argumentation levels depending on the mastery of that content and how well they are able to understand and respond to the arguments of others or produce a complete argument based on their own content knowledge.

## What are the Levels of the Mathematical Argumentation Learning Progression?

The Mathematical Argumentation learning progression has four progress variables that cut across all levels: Position, Evidence, Reasoning, and Critique.

The progress variable *Position* describes how a student understands that a proper argument must take an explicit position.

The progress variable *Evidence* describes the ways in which students provide evidence to support or counter a position.

The progress variable *Reasoning* describes how a student makes and builds a logical sequence of statements to convince the audience of their position.

The progress variable *Critique* describes the ways in which students respond to the arguments of others with clear counter arguments and are able to compare the effectiveness of two plausible arguments.

At Level 1 of the Argumentation learning progression, students may know the correct mathematical response, but do not put it in the form of an argument.

At Level 2, students see giving examples as a complete argument. They don't necessarily see how the examples need to be completed with accompanying text or diagrams that can help others to see their point of view or rationale. They may have learned that a counter example constitutes a complete proof and overgeneralize to believe that an example must also be sufficient for completeness.

At Level 3, students take a position, provide evidence, and justify their conclusions. They distinguish correct logic from flawed logic and explain the flaw. They also respond to the arguments of others with clear counterarguments. In many cases, a Level 3 response may be the top level at which we can score a written argument, if the point of the item is to explain something for another to understand.

At Level 4, students add the element of persuasion. For instance, they can compare the effectiveness of two arguments that may both have correct mathematics, but one is more logically sound. When appropriate, they are able to take into account the domain of numbers (e.g., only positive numbers, only integers) for which an argument would hold.

Below, the progression (Table 1) is shown from Level 4 (at the top) to Level 1 (at the bottom) to draw attention to the growth within the progression. Note that in this learning progression, the progress variables are not to be viewed separately, but in conjunction with one another to form a full understanding of the completeness of the argument.

### Table 1: The *Mathematical Argumentation* learning progression.

| | Title | Position | Evidence | Reasoning | Critique |
|---|---|---|---|---|---|
| Level 4 | **PERSUASION** | Takes a position and provides sufficient evidence to support the position. Conclusions are justified (same as Level 3). | Provides evidence to support a position and counter an alternative position when appropriate, with enough generalization to convince a specific audience. Distinguishes correct logic from flawed logic and explains the flaw. | Explanation of reasoning with enough evidence to convince a specific audience. Reasons inductively. Makes plausible arguments that take context into account. | Provides a critique of a position, citing evidence where appropriate. Responds to the arguments of others with clear counterarguments. Compares the effectiveness of two plausible arguments. |
| Level 3 | **PROFICIENT REASONING** | Takes an explicit position and provides examples. Conclusions are justified. | Provides evidence to support or counter a position. Uses stated assumptions, definitions, and previously established results when appropriate. Distinguishes correct logic from flawed logic. | Combination of elements into a coherent whole. Makes conjectures and builds a logical progression of statements. | Provides a critique and supports it with sound reasoning. Responds to the arguments of others with clear counterarguments, or anticipates counterarguments. |
| Level 2 | **USE OF EXAMPLES** | No explicit position, but provides examples to support an implicit position. | Provides examples, but without conclusion or generalization. Uses stated assumptions and definitions when appropriate. | Elements of reasoning but disjointed knowledge. Student may show work or solution steps for a specific problem, but does not explain how that work could generalize to other similar problems. | No critique, may have some correct mathematics, but not in the form of an argument (same as Level 1). |
| Level 1 | **LACK OF ARGUMENT** | No position. | May have correct mathematics, but not presented as evidence, lacks mathematical language. | No clear path of reasoning. | No critique, may have some correct mathematics, but not in the form of an argument. |

## What Do the Levels of the Learning Progression Look Like in Student Work?

Below is a part of a Mathematical Argumentation task, which provides an opportunity for students to demonstrate up to Level 4 of the progression in which students are asked to determine which printing company to use for the school T-shirt order. The learning progression can help educators think about what evidence each question is eliciting and anticipate the range of ways in which students might respond. For instance, in many cases a Level 3 response may be the highest level elicited if the question asks for a more simple explanation of something with a single clear answer.

*The student council at Baruch Middle School is planning to sell school T-shirts to the students in the 8th-grade class. There are about 300 students in the 8th-grade class, but the student council does not expect that everyone will buy a T-shirt. The student council is considering three different companies to make the T-shirts.*

- *EZ Tees charges $8 per shirt, and has a one-time setup fee of $200.*
- *Perfect Printing charges $4 per shirt, and has a one-time setup fee of $500.*
- *Shirts For Less charges a flat fee of $1,500 for up to 350 shirts.*

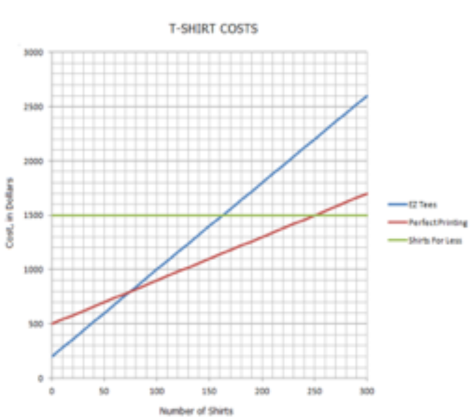
*Make a recommendation to the student council about which company they should use to order the shirts.*

**Level 1:**
***"Hello, you guys should use the EZ tees company they are cheap and can get you more with your money thank you."***

This Level 1 response does not contain a mathematical argument. It is possible that the student did interact with the mathematics, but there is nothing in the response to indicate whether this is true.

**Level 2:**
***"For 300 shirts, EZ Tees costs $2600, Perfect Printing costs $1700 and Shirts for Less costs $1500."***

This Level 2 response does contain correct mathematics [*the values for 300 shirts*], indicating that the student was engaged with the mathematical content but it is not in the form of an argument. The reader is left to decide which company to use based on a single price point [*300, which the student is told will not be the final value*]; the comparison and position is not being made by the student.




**Level 3:**
***"As you can see on the graph, Perfect Printing would be a good choice if we are not going to print all the shirts. If we print more than 250 shirts we want to go with Shirts for Less. But anything under is Perfect Printing."***

This Level 3 response takes a position [*Perfect Printing would be a good choice*], provides evidence [*via the graph*], and justifies the conclusion [*If we print more than 250 shirts we want to go with SFL, but anything under is PP*]. It shows that the student not only engaged with the mathematical content, but gave thought to the evidence that should be provided to help the student council make a decision.

**Level 4:**

***"I think you should choose Perfect Printing since it is the cheapest for 75-250 shirts. Even though EZ Tees is cheapest for <75 shirts and Shirts for Less is cheapest for >250 shirts, I still think you should use Perfect Printing because a lot of people will want the shirts but not everyone, so it will probably be Perfect Printing that is the cheapest."***

This Level 4 response not only takes a position [*I think you should choose Perfect Printing*], provides evidence [*numerical ranges for which each company would be the cheapest*], and justifies the conclusion [*I still think you should use PP because a lot of people will want the shirts but not everyone, so it will probably be PP that is the cheapest*], but it also anticipates potential counter arguments [*<75 and >250 shirts*] and provides a response to those counterarguments to convince a reader of the conclusion.

## How Can We Help Students Learn?

Teachers can use the Mathematical Argumentation learning progression in the following ways:

- As a guide to anticipate and interpret student thinking, including selecting instructional materials or developing classroom assessments (e.g., Are the students able to provide a counterargument? Are they able to find flaws in an argument? Can they convince other students of their conclusions?)
- To develop hypotheses about what students do and do not yet understand, based on evidence of student thinking
- To determine next steps to support emerging understanding

Teachers can engage with these and other practices individually or with colleagues. Examining student work to understand how it addresses the standards while using the learning progression to interpret more and less sophisticated responses can support further instruction. Planning together how to give feedback to students or identify next instructional steps for students who are at different levels of the learning progression can also be a useful professional learning experience.

Students may also find the learning progressions useful with some translation into more student-friendly language and exemplars to illustrate reasoning at different levels of the progression.

## For Additional Insights

Other relevant learning progressions are Mathematical Modeling.

## For More Information

For further reading, see Rumsey, C., & Langrall, C. W. (2016). Promoting mathematical argumentation. *Teaching children mathematics*, 22(7), 412–419.

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