# *Winsight*® Assessment Mathematics Learning Progressions

## LINEAR FUNCTIONS

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# Mathematics Learning Progressions: Linear Functions

## What is the Linear Functions Learning Progression?

The learning progression for Linear Functions focuses primarily on how the understanding of linear functions develops and deepens in middle school grades (Table 1). The transition from Level 2 through Level 4 describes development from recognizing patterns of change to understanding linear relationships to comparing linear functions. Students may use linear functions when they encounter real-world problems (with either positive or negative slope), such as determining the cost of a taxi ride that has an initial pick-up charge and then a cost per mile, or determining how many hours of part-time work are needed to meet short-term and long-term savings goals. Students can communicate relevant information in different ways, otherwise known as representations, such as tables, graphs and equations. Because students need to develop both an understanding of change and how to represent linear functions, we have highlighted these as progress variables, or main ideas, in the learning progression.

## Connecting the Linear Functions Learning Progression to the Common Core State Standards

Linear functions connects to several Common Core State Standards for mathematics for grades 6 to 8. Middle grades Linear Functions is directly addressed in CCSS-M 8.F 2-4, but relationships between quantities is also covered in CCSS-M 7.RP 2. It is important for teachers to understand the connection between proportional relationships (represented by $y = mx$) and linear functions (represented $y = mx + b$, where $b \neq 0$) and use this information to inform instruction.

## What are the Levels of the Linear Functions Learning Progression?

The Linear Functions learning progression has two progress variables that cut across all levels: *Concept of Change* and *Integration of Representations*.

The progress variable *Concept of Change* describes how a student develops understanding of the relationship between two quantities.

The progress variable *Integration of Representations* describes the ways in which students understand and learn to use different function representations such as tables, graphs or equations.

The Learning Progression (Table 1) is shown from Level 5 (at the top) to Level 1 (at the bottom) to draw attention to the growth within the progression. Student performance at higher levels assumes that the student has the knowledge and skills at lower levels. For example, if a student can compare linear functions, we assume that he or she understands slope.

## Table 1: The Linear Functions learning progression.

| | Concept of Change | Integration of Representations (i.e., verbal, numeric, spatial and symbolic) |
|---|---|---|
| **Level 5 Changing Change** | The student recognizes and interprets a continual changing slope (i.e., the slope is different at different points) and understands that linear functions describe relationships of constant change whereas non-linear functions do not. | The student fully integrates numeric, spatial and symbolic representations, including those of changing change. |
| **Level 4 Comparing Rates of Change** | The student:<br>• compares linear functions (i.e., compares constant changes and intercepts)<br>• translates and/or rotates linear functions by changing the y-intercept and/or slope, understanding implications of translation and rotation for the function<br>• recognizes and interprets functions of split domain (i.e., piece-wise functions) | The student:<br>• coordinates information from different given representations<br>• translates between all representations of linear functions, where the student constructs at least one representation (e.g., the student is given an equation and must construct a graph). This includes translation of verbal and tabular representations to equations |
| **Level 3 Constant Change** | The student:<br>• understands rate as a relationship (i.e., constant change) b/w x and y<br>• correctly interprets the magnitude and direction of positive and negative slope<br>• for non-proportional linear relationships (e.g., $y = 3x - 5$), generates a rule for a given table or a table for a given rule<br>*Difficulties:* The student cannot work with more than one function at a time. | The student:<br>• works with the symbolic form of functions with the form $y = mx$ or $y = mx + b$ (or with different forms of this function)<br>• can translate an equation into a verbal, tabular or graphic representation<br>• can translate from a table to a graph and vice versa<br>• translates and/or rotates a linear function by changing the y-intercept and/or changes the slope of a function (may or may not have a complete understanding of what this means in context)<br>• interpolates or extrapolates a line graph<br>• interprets the meaning of the line in context<br>*Difficulties:* The student does not integrate or translate between all representations. |
| **Level 2 Mutual Change** | The student:<br>• recognizes patterns and completes missing numbers in a table or in paired sequences<br>• for proportional linear relationships (e.g., $y = 3x$), generates a rule for a given table or a table for a given rule<br>*Difficulties:* The student does not understand rate as a relationship between x and y (even if the student can calculate rate from two points) and/or does not understand slope or y-intercept. | The student:<br>• plots and interprets points on a coordinate plane<br>• plots basic tables of changing x, y on a coordinate plane, while noticing the mutual change<br>*Difficulties:* The student does not yet understand the meaning of a line connecting the plotted points and/or cannot link a spatial representation with the symbolic one. |
| **Level 1 One-Dimensional Change** | The student:<br>• recognizes or completes patterns<br>• generates a rule for a given pattern (i.e., a one-dimensional change) or a pattern for a given rule<br>*Difficulties:* The student cannot yet work with two-dimensional change (e.g., extrapolate or interpolate pairs of values in a table; cannot generate a rule given a table or a table given a rule). | The student interprets the relative size of bars in a bar graph.<br>*Difficulties:* The student cannot fill in missing bars in a bar graph. |

At Level 1 of the Linear Functions learning progression, students can understand bar graphs and may understand or identify a rule or a pattern within one sequence of numbers only. While these knowledge and skills may be taught before the middle grades, they are important precursors to understanding linear functions.

Limited use of multiple representations begin at Level 2, where students can represent points on a coordinate plane and can plot values from a table onto a coordinate plane. At this level, the concept of change expands to understanding mutual change, or change occurring simultaneously in two related sequences of numbers in an input-output table or on the coordinate plane.

By Level 3, numeric, tabular and graphic representations can be used along with symbolic representations, and students begin to understand the concept of constant change. At Level 3, rate is understood as a relationship between x and y (i.e., constant change) and is connected to the concept of slope. A common misconception at this level is that students will assume that some situations that are non-linear can be modeled using a linear function (i.e., an overgeneralization).

Students at Level 4 interpret and translate between multiple types of representations, and can use their knowledge of mutual change to compare different changes (e.g., comparing slopes, comparing two or more linear functions).

By Level 5, students understand the more sophisticated concept of changing slopes (i.e., changing change). This understanding forms the basis for higher-level mathematics, such as non-linear functions and derivatives and is a transition to other non-linear learning progressions (Concept of Function, Quadratic Functions and Exponential Functions).

## What Do the Levels of the Learning Progression Look Like in Student Work?

Below is an example of a linear functions task, targeting Levels 2–4 of the progression. The learning progression can help educators think about what evidence each question is eliciting and anticipate the range of ways in which students might respond.

*It costs $400 to reserve a recording studio, plus $200 for each hour that you spend recording in the studio.*

*1. Use the information above to complete the table below.*

*Hours, x* *Dollars, y*

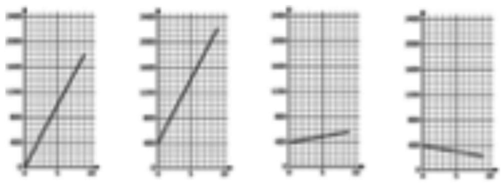
*0*

*4*

*1400*

A student who performs at Level 2 should be able to use the verbal information to complete the table in Question 1, although some students may need some additional scaffolding in the form of a table that shows costs for one, two and three hours rather than jumping to calculate the cost for four hours.

*2. Which graph below correctly shows the relationship between y, the total amount of money you spend, and x, the number of hours you spend recording?*



*3. Explain how you found your answer to question 2.*

A student who is at Level 3 should be able to do everything at Level 2 and also identify the second graph as the correct representation of the values in the table. This student, however, would likely struggle if s/he were asked to provide an equation to represent the line or the table.

Responses to Question 3 will vary across students, with some students mentioning only one of the two critical aspects of the answer. An incomplete answer could be followed up with additional probing to determine whether it is evidence of not fully understanding, or whether, with support, the student can demonstrate their understanding. The student explanations below represent a range of understanding that map to different levels of the learning progression (although a determination of the most appropriate level of a student's understanding would not come from a single question).

- ***All I had to do was check to see which one had $1,200 for 4 hours and $1,400 for 5 hours.*** This response shows evidence of Level 2 thinking in which the student is able to plot and interpret specific points on a coordinate plane using the data in the table to identify the correct answer, but is not describing the graph in terms of slope and intercept.
- ***I found my answer by looking at the graph and thinking how much you would have to pay if you didn't spend an hour recording and that was $400 just to rent the place and with the first graph it shows that it would be $0. It could not be the 4th graph since the cost does not go down with time. I also know that it costs $200 per hour, so you add 200 to the y value every time the x value goes up by one, so it had to be the second graph.*** This response shows evidence of Level 3 thinking in which the student is able to use the concept of slope and intercept to justify the correct answer.
- ***.. because the y intercept is 400 and it is a positive slope, so the first one and the last one are incorrect. The slope is 200 since y = 200x + 400, so it has to be the second graph.*** This response shows evidence of Level 4 thinking, since the student used another representation to compare functions, providing the correct equation to represent the situation.

## How Can We Help Students Learn?

Teachers can use the Linear Functions learning progression in the following ways:

- As a guide to anticipate and interpret student thinking, including selecting instructional materials or developing classroom assessments (e.g., Are the students able to interpret and create multiple representations of linear functions? Are they able to compare two or more functions? Can they explain the meaning of slope?)
- To develop hypotheses about what students do and do not yet understand, based on evidence of student thinking
- To determine next steps to support emerging understanding

Teachers can engage with these and other practices individually or with colleagues. Examining student work to understand how it addresses the standards while using the learning progression to interpret more and less sophisticated responses can support further instruction. Planning together how to give feedback to students or identify next instructional steps for students who are at different levels of the learning progression can also be a useful professional learning experience.

Students may also find the learning progressions useful with some translation into more student-friendly language and exemplars to illustrate reasoning at different levels of the progression.

## For Additional Insights

Other relevant learning progressions are Proportional Reasoning (many proportional reasoning problems can be represented as a linear function with a zero intercept), Concept of Function (students develop an understanding of the concept of a function and how a linear function is different from and similar to other types of functions), Quadratic and Exponential Functions, Mathematical Modeling (identifying when and how a situation can be modeled using a linear function), and Argumentation (develop and critique arguments about a situation using one or more linear functions).

## For More Information

For further reading, see Graf, E., & van Rijn, P. (2016). Learning progressions as a guide for design: Recommendations based on observations from a mathematics assessment. In S. Lane, M. R. Raymond, & T. M. Haladyna (Eds.), *Handbook of test development*, 165–189.